Progress in Non-Fourier Heat Conduction at Small Scales

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Abstract. Classical Fourier law can accurately describe most heat conduction problems. But for ultrafast heat conduction process and micro/nanoscale heat conduction problems, non-classical Fourier (non-Fourier) effect may become dominated. The paper gives a review on the current progress on non-Fourier heat conduction in engineering. It includes basic concept, physical models, thermal relaxation effect, and related experiments. Also introduced are the solution methods of non-Fourier heat conduction equations, including closed-form solution, finite difference method, finite element method, molecular dynamics simulation, variational method, and other hybrid methods. Some challenging issues are discussed at the conclusion of the paper.

Introduction

Early in the 19th century, Maxwell’s kinetic theory of gases gives the embryonic form of temperature wave theory. During the next 100 years, the research on heat wave theory got great development by the efforts of many theoretical physicists and experimental physicists. Tisza and Londau [1] proposed heat exists as wave-form in assumption that heat propagates at finite velocity. In 1944, Peshkov [2] proved the existence of heat-form wave. In 1953, Morse and Feshbach [3] considered the general characteristics of non-Fourier heat conduction and Fourier heat conduction, and put forward the general Fourier law. In 1958, Cattaneo [4] and Vernotte [5] respectively proposed the non-Fourier heat conduction constitutive equations based on the non-Fourier heat conduction law and local energy balance equation. With the development of transient heat flow, high density heat input, strong radiation heating and cooling technology (e.g. ultrashort pulse laser heating, cryogenics), people began to research non-Fourier effect of metal thin film, semiconductor material, superconductive film, porous material, and biological tissue in ordinary temperature conditions.

In general view, no matter what pathogenesis is, the heat conduction process should obey the Fourier law, which is based on the hypothesis of infinite heat propagation velocity. For the non-steady conventional heat conduction process of slow heat propagation or steady heat conduction process of long thermal action time, there is no doubt about the validity of Fourier law. But as for non-steady heat conduction process that extremely fast heat transfer of mass transfer, e.g. the heat (mass) transfer problem in extremely high (low) temperature conditions, the heat (mass) transfer problem with extremely great temperature gradient, the heat (mass) transfer problem in micro-time scale or micro-space scale conditions, the finite velocity of heat propagation must be considered. Hence, there must presents some thermal physical characteristics different from conventional heat transfer progress. The heat transferring effect deviating from or disobeying Fourier law in extremely heat (mass) transferring conditions is referred as non-Fourier heat conduction effect, also called non-classical heat conduction effect.

Various non-Fourier models

Various models from different physical aspects have been constructed. Next will demonstrate several widely accepted non-Fourier models – Single phase lagging model, Dual phase lagging model, Parabolic Two-step model, Hyperbolic Two-step model.

Tzou [6] considered the finite velocity of heat ensures time lagging between the propagation of
heat flux vector and the forming of temperature gradient, introducing relaxation time $\tau_0$ \[7\], and proposed Single Phase Lagging Model. The mathematical expression is

$$q(r, t + \tau_0) = -k\nabla T(r, t)$$ \hspace{1cm} (1)

The Taylor expansion about time $t$ is 

$$q(r, t + \tau_0) = q(r, t) + \tau_0 \frac{\partial q(r, t)}{\partial t} + o(\tau_0^2).$$

This coupled with with local energy equation 

$$s(r, t) = \rho c_p \frac{\partial T(r, t)}{\partial t} + \nabla \cdot q(r, t)$$

and neglecting high-order terms, gives the SPL model expression

$$\rho c_p \left[ \frac{\partial T(r, t)}{\partial t} + \tau_0 \frac{\partial^2 T(r, t)}{\partial t^2} \right] = k\nabla^2 T(r, t) + \left[ s(r, t) + \tau_0 \frac{\partial s(r, t)}{\partial t} \right]$$ \hspace{1cm} (2)

Tzou \[8\] considered the micro-structures influence the macro-features, introducing phase lagging both to the heat flux vector and temperature gradient, and proposed Dual Phase Lagging Model describing the non-Fourier heat conduction phenomenon. The mathematical expression is described as

$$q(r, t + \tau_q) = -k\nabla T(r, t + \tau_T)$$ \hspace{1cm} (3)

Heat flux vector lagging $\tau_q$ reflects the heat wave-form characteristics, and temperature gradient lagging $\tau_T$ reflects the phonon-electron interaction. Doing Taylor expansion to eq. \[3\] about time $t$, and neglecting the high-order terms, one obtains

$$q(r, t) + \tau_q \frac{\partial q(r, t)}{\partial t} \approx -k[\nabla T(r, t) + \tau_q \nabla T(r, t) / \partial t]$$ \hspace{1cm} (4)

Coupling with energy conservation equation, the hyperbolic heat conduction equations about temperature $T$ or heat flux vector $q$ can be derived.

Parabolic Two-step model \[9\] are extensively used in research of micro-scale heat conduction, e.g. the ultrashort pulse laser radiation process, involving energy transportation that electron transferring to lattice in picosecond-scale. During the non-equilibrium heating process, the continuous energy flow equations describing heat electron transferring to lattice as

$$C_e(T_e) \frac{\partial T_e}{\partial t} = \nabla \cdot \left[ k_e(T_e, T_l) \nabla T_e \right] - G(T_e - T_l) + S, \quad C_l \frac{\partial T_l}{\partial t} = G(T_e - T_l) \hspace{1cm} (5)$$

where $e$-electron, $l$-lattice, $C$-volume heat capacity, $G$-electron-lattice coupling factor, $S$- external heat source

When heat response time is far less than the relaxation time of free electron (the average time electron changing state), the Parabolic Two-step model is not sufficient to describe the continuous energy flow during non-equilibrium thermodynamics process. In order to describe the non-equilibrium behavior between electron and lattice, Qiu and Tien \[10\] proposed more rigorous Hyperbolic Two-step model based on quanta-mechanical and statistics theory. First the photons provided by external source improve the temperature of electron gas as

$$C_e \frac{\partial T_e}{\partial t} = -\nabla \cdot \bar{q}_e - G(T_e - T_l) + S, \quad \tau_e \frac{\partial \bar{q}_e}{\partial t} + \bar{q}_e = -k_e \nabla T_e \hspace{1cm} (6)$$

Then through the action between electron and photon heating the metal lattice as

$$C_l \frac{\partial T_l}{\partial t} = -\nabla \cdot \bar{q}_l + G(T_e - T_l), \quad \tau_l \frac{\partial \bar{q}_l}{\partial t} + \bar{q}_l = -k_l \nabla T_l \hspace{1cm} (7)$$

In addition, there exist some other model such as heat wave model based on entropy production theory \[11\], random discontinuous diffusion model of heat propagation \[12\], phonon heat transportation model based on Boltzmann theory \[13\], Cattaneo model with correctional boundary conditions \[14\], pure phonon scattering model \[15\], three-step model of introduced displacement lagging \[16\], and non-equilibrium heat transportation model of non-homogeneous inner structure medium \[17\].

**Demonstration on non-Fourier heat conduction effect**
The validity of Fourier law is no doubt for the conventional heat transfer process. However, for the heat (mass) transfer problems in extremely low (high) temperature conditions or the heat (mass) transfer process during micro time (space) scale, the non-Fourier effect must be considered.

The existence of non-equilibrium state during thermodynamics changing process is the main cause to the heat finite velocity phenomenon. The heat finite velocity makes heat disturb and heat response lagging, i.e. there existing heat relaxation time $\tau_0$. According to molecular collision theory [18]:

$$\tau_0 = \frac{\alpha}{c_h^2}$$  \hspace{1cm} (8)

where $\alpha$ is thermal diffusion rate, $c_h$ is the velocity of heat wave-form. When heat velocity tends to infinity, $\tau_0=0$, $c_h \rightarrow \infty$ is the situation Fourier law describes.

Maurev [19] and Francis’s [20] research demonstrate that heat relaxation time varies with different material characteristics. In conventional temperature, relaxation time of metal ranges from $10^{-12}$s to $10^{-10}$s, gas from $10^{-10}$s to $10^{-8}$s, liquid and dielectric are intervenient between metal and gas. For porous capillary substance, the relaxation time is $10^3$ to $10^7$ times larger than metal. And that for porous medium, biological tissue, the relaxation time reaches several ten seconds.

For multi-layered material, it is necessary to consider the interface heat resistance effect. Lor [21] adopted Radiated Boundary Model based on Phonon Scattering Model, and research on the interface thermal effect of two-layered material. Pulsed incident energy acts on the layer 1 surface, and released thermal energy, generating temperature wave within skin depth. Then emanate into layer 2. By employing radiation-boundary-condition model, the continuity of interface heat flux is

$$q_{i1} = q_{2i} = \frac{2\pi k_i \Gamma}{\hbar^2\nu} \left( \frac{\pi^4}{15} (T_{i1}^4 - T_{2i}^4) \right)$$  \hspace{1cm} (9)

where $\Gamma$ is a function of the material properties of the two medium in contact.

Liu [22] based on hyperbolic microscopic two-step model, and analyzed the non-equilibrium thermal behavior in multi-layer metal film. The different thermal physical properties of different material cause mathematical complexity and difficulty. Results demonstrate the hyperbolic nature of heat transfer that thermal pulses travel back and forth in electron gas at finite propagation speed. Lv [23] did research on micro-geometry structure effect of porous material radiated by short pulsed laser. Vermeersch [24] adopted complex thermal impedance with thermal step response to analyze the non-Fourier heat conduction characteristic of nano-scale semiconductor matrix, and the results indicate the wave nature of non-Fourier heat conduction process.

Tabrizi [25] proposed technology based on frequency response to measure lagging time in Dual Phase Lagging model and T wave model. First he defined thermal impedance, a complex function of source frequency, lagging time, and thermo physical properties as

$$Z_{th} CV = \frac{(1 + i\tau_q \omega)}{km_{CV}}, \quad m_{CV} = \sqrt{\frac{i\omega - [\omega(\tau)]_q \omega}{\alpha}}$$

$$Z_{th} DPL = \frac{(1 + i\tau_q \omega)}{km_{DPL}(1 + i\tau_q \omega)}, \quad m_{DPL} = \sqrt{\frac{i\omega - [\omega(\tau)]_q \omega}{\alpha(1 + i\tau_q \omega)}}$$  \hspace{1cm} (10)

where $Z_{th}$ is thermal impedance, $\omega$ is frequency of imposed heat flux, $k$ is thermal conductivity, $\alpha$ is the thermal diffusivity.

Then through the measurement of thermal impedance value obtained the lagging time value. This method does not have computational error, and the accuracy of determined lagging time mainly depends on measuring technology and measuring device.

Ordonez-Miranda [26] considered a finite layer attached to a semi-infinite layer of different material excited by a modulated heat source. They considered thermal physical characteristics of two parts, obtained the amplitude and phase functions about frequency of source. Through measuring the
value of frequency for which the amplitude has a maximum or minimum, used the equation \[13\] to determine \(\tau\),

\[f_n = \frac{1}{2l} \sqrt{\frac{\alpha}{\tau}} \begin{cases} n + \frac{1}{2}, A = A_{\text{max}} \\ n, A = A_{\text{min}} \end{cases}
\] (11)

where \(A\) is the amplitude and \(n=1, 2, 3\...\)

**Experimental research of non-Fourier heat conduction**

The earliest non-Fourier heat conduction experiment was carried on low-temperature conditions. During 4th decade of last century, Tisza and London proposed two-fluid model in the research of super-liquid Helium, and predict that there existing heat wave-form in the propagation of finite velocity in liquid-HeliumII, i.e. the second sound wave. In the year of 1944, the Soviet Russia scientist Peshkov utilize pulsed electric-heat generator to generate the second sound wave, and measure that at the temperature of 1.4K, the heat propagation speed in liquid-Helium is about 19m/s. Afterward, there is little essential evolvement on non-Fourier heat conduction experiment. Until 80th decade last century, with the development of short pulsed laser technology and the emergence of new-type temperature measurement technique, it has some progress on non-Fourier experiment research. So far, non-Fourier heat conduction experiments still grope for its direction.

Qiu and Tien [27] did research on the energy transferring process that femosecond pulsed laser radiated gold-chromium multilayered thin film, and compare experiment date with the result PTS model predicts, fitting well. Jiang [28] proposed an experiment of a transient pulsed laser heating to porous material under normal conditions, experimental apparatus. The result demonstrates that only when heat disturbance are sufficiently strong, non-Fourier effect could be watched in extremely short time. Based on one-dimensional Fourier heat conduction model, Yuan [29] analyzed the temperature rising characteristic of 30CrMnSiA steel and LF6M aluminum gold coating radiated by repeating frequency laser. Zhang [30] considered the laser igniting process of energy-containing material as powerful transient heat process, and simulate the igniting process of gunpowder using numerical method based on one-dimensional spherical coordinate system non-Fourier heat conduction equation. Hao [31] extended the use of existing temperature two-step model to high electron temperatures by using full-run quantum treatments, including the electron heat capacity, electron relaxation time, electron conductivity. Then through introducing artificial viscosities and adaptive time steps, hyperbolic two-step heat conduction equations are numerically solved by an accurate and stable forward-difference scheme. And the histories of electron temperatures distribution about time of a 200nm gold film heated by a laser pulse with \(t_p=0.14ps, J_o=4700J/m^2\) are obtained. Mitra et al. [32] experiment indicate that heat takes finite time to reach characteristic point in form of wave, and violate the situation Fourier law depicts, demonstrating the existence of non-Fourier effect in biological material.

**Theoretical development of non-Fourier heat conduction equation and solution methods**

Fourier law describes the relation between heat flux vector and temperature distribution as the constitutive equation of heat conduction theory. The corresponding mathematical expression is a parabolic partial-differential equation \(\frac{\partial T}{\partial t} - \alpha \nabla^2 T = 0\). This law assumes that heat acts as diffusion behavior in infinite velocity.

Morse and Feshbach consider the finite velocity of wave-form heat and proposed correctional Fourier heat conduction law as \(q = -\nabla T - \tau_0 \frac{\partial q}{\partial t}\). This equations, coupling with the energy equation (neglecting generating source) \(\rho c \frac{\partial T}{\partial t} + \nabla q = 0\), yields the hyperbolic heat conduction differential expression:
\[
\tau_0 \frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t} = \alpha \nabla^2 T
\]

For Dual-phase-lag heat conduction model, introducing phase lagging both to heat flux vector and temperature gradient, the correctional non-Fourier heat conduction law can be expressed as

\[
q = -k \nabla T - k \tau_T \frac{\partial \nabla T}{\partial t} - \tau_q \frac{\partial q}{\partial t}
\]

where \( \tau_T \) and \( \tau_q \) are, respectively, phase lagging time of temperature gradient \( \nabla T \) and heat flux vector \( q \). Coupling with energy equation, derived heat conduction equation in form of temperature \( T \) or heat flux vector \( q \) is

\[
\frac{\partial T}{\partial t} + \tau_q \frac{\partial^2 T}{\partial t^2} = \alpha \nabla^2 T + \alpha \tau_T \nabla \left( \alpha \nabla T \right) = \alpha \nabla \left( \alpha \nabla q \right) + \alpha \tau_T \nabla \left( \alpha \nabla q \right) / \partial t
\]

Dual Phase Lagging model expressions are relatively more comprehensive, but as a result of the format complexity and solving difficulty, having limited application.

**Closed-form solution** About the analysis solution of non-Fourier heat conduction equation, most are limited to trial function or build up function analysis-form based on mathematical experience. Hosein Shokouhmand [33] adopted trial function method combined with optimization allocation rule to derive the temperature distribution of one-dimensional varying heat conduction coefficient problems. First through Laplace transform, heat conduction equation and boundary conditions are transformed from time-domain into Laplace-domain. Then utilizing allocated optimization rule, applying trial function method derived the polynomial function contained unknown coefficient and satisfying boundary conditions. Last the results are transformed into physical space through Laplace inversion transform, obtaining temperature distribution. R.X. Cai [34] put forward four set analytical solutions satisfying spherical coordinate governing equation and different boundary conditions aiming at IC chip. Fangming Jiang [35] considered the hyperbolic heat transfer process in a hollow sphere with two boundary conditions subject to sudden temperature changes and obtained analytical expression of temperature profile by means of integration transformation. Monteiro [36] adopted Generalized Integral Transform Technique (GITT) to solve the thermal wave propagation problem of finite slab. First applied GITT method to transform hyperbolic heat conduction equation into second-order ordinary differential equation system in time domain, then used finite volume – Gear’s method to solve the obtained system numerically, and give the local temperature distribution and average temperature distribution of different dimensionless thermal relaxation time value and Biot numbers.

**Numerical solution** For heat conduction problems with complicated geometry boundary conditions or varying thermal physical properties, closed-form solution is hardly got. Hence, numerical methods are widely used, including Finite Difference Method (FEM) based on Eulerian theory, Finite Element Method, Boundary Element Method, SPH (Smoothed Particle Hydrodynamics) method based on Lagrangian fluid theory, and modern Molecular Dynamics, e.t.c. The drawback solving hyperbolic heat conduction equation by numerical methods is there existing oscillation in sharp discontinuity vicinity.

**Finite Difference Method (FDM)** Applying FDM to partial differential equation, firstly partition the solution domain into grids, then instead the differential equation with difference equation in nodes. The solution accuracy would be higher if partitioning grids more densely. Finite Element Method can solve quite complicated mechanics problems, especially having great advantage in fluid mechanics problem based on Eulerian system.

F.M. Jiang [37] considered adopting SPH method to solve heat conduction problem and fluid flow problem of composite medium with complex microstructure. Q.M. Fan [38] put forward Dual Reciprocity Bound Element Method (DRBEM) combined with Laplace transform technology to solve hyperbolic heat conduction problem. First moving away time term by Laplace transform, and transforming corresponding initial condition with boundary condition into Laplacian domain. Then
according to the theory of dual reciprocity, transform governing equation and boundary condition into pure integral equation. Last transform the result into physical space using Laplace inverse-transform method proposed by Stehfest in 1970. Ramos [39] proposed the finite difference method of second-order accurate space discretization with time-linearization on nonlinear term to discuss one-dimensional hyperbolic heat conduction problem. And compared the result with obtained analytical result demonstrate that the accuracy of time-linearized method will increase as decreasing time-step and grid spacing. Dai [40] based on hyperbolic two-step model and solved the temperature distribution of three-dimensional microsphere radiated by ultra short pulsed laser. Niu [41] developed new finite difference term, introducing LP norm to obtain energy evaluation, and analyzed the relation between the temperature and thermal physical characteristic of two-layered micro scale metal film with nonlinear interfacial conditions. Ciegis [42] constructed explicit and implicit Euler difference scheme to develop robust and efficient algorithm, both for parabolic and hyperbolic heat conduction problem. Then did stability analysis by different methods (e.g. the maximum principle, spectral analysis or energy estimate), and applied Lax Theorem to demonstrate the well convergence of the method. 

Yu [45] based on Single Phase Lagging model, and adopted finite element method combined with Newmark-β method to research on two-dimensional planar heated by local pulsed heat flux. First introducing dimensionless parameters derived the dimensionless expression of hyperbolic equation. Then applying weighted residual method to discretize the governing equation, and choosing appropriate β value with Newmark-β linear acceleration method to solve second-order linear differential equation system. Loh [46] put forward the method combined asymptotic wave evaluation (AWE) technology with FEM. Compared with conventional iterative algorithm, AWE calculation speed is at least three magnitudes faster. But due to the ill-conditioned characteristics of AWE moment matching process, there may generate instable response. Miller [47] proposed spacetime discontinuous Galerkin finite method to solve hyperbolic heat conduction problem. First according to Stokes theorem and Localization theorem to derive the governing equation, meanwhile partitioning boundary condition and calculating target flux. Through discrete weighted residual method, complete spacetime discontinuous Galerkin formulation. Then applied causal advancing-front meshing procedure to enable a patch-wise solution procedure with linear complexity in the number of spacetime element, and adopted h-adaptive scheme with special shock-capturing operator to accurately solve the sharp solution features in both space and time.

Molecular Dynamics Simulation Molecular Dynamics Simulation considers the system as molecular set with some characteristics, and regard molecular (microscopic particles) as basic research object. MD simulation first apply Newtonian Mechanics or quantum mechanics method to research the motion law of microscopic particles. Then through the configuration integral of system, derived the macroscopic characteristics and fundamental law. The advantage of MD simulation lies

**Finite Element Method (FEM)** Finite Element Method is widely used in heat conduction problems involving complicated geometry boundary conditions. FEM start from the differential equations’ equivalent integral form, and commonly adopts weighted residual method to construct several approximated solution, e.g. allocated point method, least square method, Galekin method. 

Du [48] based on time-discrete precise integration method proposed by W.X. Zhong, and presented the solution algorithm of hyperbolic heat conduction equation discretized by FEM in space. Applying the algorithm to two-dimensional circular problem, after calculation and result analysis, demonstrate the validity and high accuracy to non-Fourier heat conduction problem.
in revealing nature of macroscopic appearance through the understanding and comprehension to microscopic phenomenon. The disadvantage lies in that the program procedures are quite complex and memory occupancy large.

The practical steps of molecular dynamics simulation could be divided into four steps:

1. Set the adopting model of MD simulation

\[ U(r) = 4\varepsilon \left( \frac{\sigma}{r} \right)^{12} - \left( \frac{\sigma}{r} \right)^{6} \]  \hspace{1cm} (15)

where \( \varepsilon \) is potential well constant, \( \sigma \) is equilibrium constant.

2. Given the initial conditions

3. The calculation process of tending to equilibrium

To let system achieving equilibrium, there needs an equilibrium process in simulation. During the non-equilibrium process, adding up or moving away energy until system possess acquired energy.

4. Calculus of macroscopic physical quantities

\[ \bar{E}_k = \frac{1}{n-n_q} \sum_{i=1}^{n} \sum_{r=1}^{N} \left( \frac{p_i^2}{2m} \right) \quad \bar{U} = \frac{1}{n-n_q} \sum_{i<j} \sum_{r=1}^{N} U \left( r_{ij} \right) \]  \hspace{1cm} (16)

Predicted correctional method is the general method in MD simulation, and the basic idea is Taylor expansion, other algorithm including Verlet algorithm, Velocity Verlet algorithm, Leap-Frog algorithm, Beeman’s algorithm, etc.

Liu [49] used Classical Molecular Dynamics Simulation to investigate non-Fourier heat conduction effect in both pure argon thin films and film with vacancy defects subject to a sudden temperature increase or heat flux at one surface, and showed the existence of mechanical waves when the film was suddenly heated and the wave nature of heat propagation.

First used Lennard-Jones pair potential to describe the interaction between atoms, then given the initial temperature of 10K by choosing their velocities based on the Maxwell distribution. And defined the temperature of each layer as

\[ T = \frac{1}{3Nk_B} \sum_{i=1}^{N} m_i v_i^2 \]  \hspace{1cm} (22)

where \( N \) is the number of atoms in the layer.

**Variational Method** Transforming a physical problem using Variational method into seeking extreme value of functional is called the variation principle. Variation principle are widely used in physics especially in mechanics, e.g. the famous principle of virtual work, principle of minimum potential energy, principle of minimum complementary potential energy, Hamilton principle. In practical application, the accurate closed-form solution is usually seldom obtained. Hence, most calculations are based on approximate methods, including Ritz method, Galekin method, Kantorovich method.

Saleh [50] adopt Laplace transform combined with variation calculus to solve non-Fourier heat conduction equation of homogeneous isotropic medium. And applying first-order approximate function and second-order approximate function derived the Laplacian domain temperature distribution of one-dimensional heat conduction problem. Through approximate convergent function and Taylor series expansion derived the inverse Laplace transform solution, and compared with obtained analysis solution demonstrate that for small value dimensionless time, variation calculus could achieve sufficient accuracy.

**Extra Methods** General numerical method happen strong oscillations in sharp discontinuity of solution domain, and many scholars put forward many methods to suppress oscillations. Chen [51]
adopted hybrid Green function method—Laplace transform combined with Green function and ε—algorithm acceleration to solve hyperbolic heat conduction problem. First deriving the dimensionless expression of three-dimensional Single Phase Lagging heat conduction model, and applying Laplace transform to move away dimensionless time term. Then introducing auxiliary Green function, through variables separation method derived the Laplacian domain solution. And applying numerical inverse Laplace transform technology with ε—algorithm acceleration to obtain the dimensionless temperature distribution in physical space. Huang [52] based of Dual Phase Lagging model, and adopted Conjugate Gradient Method (CGM) to solve inverse hyperbolic heat conduction problem of unknown source. Through evaluating offset error and random error, demonstrate the effectiveness of conjugate gradient method. Lo [53] adopted hybrid differential transform technology combined with control volume method to solve hyperbolic heat conduction problem. Compared the result with Tsai [54] analysis solution demonstrate that choosing appropriate power exponent value of shape function can effectively suppress numerical oscillation. Chen [55] first applied Laplace transform to move away the time terms, then used control volume method to derive the discrete expression of transform equation. Last combine curve-fitting technology with least-square method to evaluate the unknown boundary conditions of one-dimensional hyperbolic inverse heat conduction problem. Seaid [56] adopted classical Euler-Lagrange method to analyze coupling parabolic-hyperbolic equations. Firstly adopted the modified method of characteristics belonging to the class of fractional-step procedures to accurately solve the hyperbolic equations, then constructed Eulerian explicit scheme with a large stability region to discretize the parabolic equations in space, meanwhile used Runge-Kutta Chebyshev scheme for the time integration. Last presenting two examples about coupled convection-radiation flow and coupled conduction-radiation problem, demonstrates that using reasonably large steps, the Eulerian-Lagrangian method reproduces the corresponding flow patterns and accurately captures the flow structures with very little numerical diffusion, even after long time simulations.

Conclusions

In engineering applications there existing broad non-Fourier phenomenon. For example, the metal rapid solidification process, the thermal stability control of superconducting coil, the fusion reactor, the synchrotron source transferring high-energy X-ray, the laser diagnostics and laser processing, the temperature controlling of high-power laser weapon reflector, super rapid freezing and thawing process of body organ in biomedical engineering, the cooling of superconductor electronic device (e.g. nano-scale temperature sensor, nano-scale metal film thermal inductor).

In modern society, with the development of (ultra)short pulse laser technology, nuclear technology, micro-scale heat transfer technology, more and more scholars and experts are interested in the research of non-Fourier heat conduction phenomenon.

So far, the problems of non-Fourier heat conduction have the following challenging issues:

1. Some researchers put forward the question that hyperbolic heat conduction equation does not obey the second thermodynamics law. For example, research on non-equilibrium entropy under Dual Phase Lagging model suggested that adopting equilibrium-entropy classical form cannot describe the entropy production, leading to the violation of the second thermodynamics [57].

2. There is great difficulty in obtaining analysis solution of hyperbolic heat conduction problems involving complicated boundary conditions.

3. As for numerical methods, the calculation results rely largely on computational methods. In most cases, the results show numerical oscillations. Hence, an appropriate method is critical in obtaining the correct results.

4. Experiment research on non-Fourier heat conduction is still rare.

References